Equations and Inequalities

Algebra 2

Chapter 1

- This Slideshow was developed to accompany the textbook
  - *Larson Algebra 2*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

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1.1 Apply Properties of Real Numbers

Real Numbers

Rational Numbers – Can be written as fractions

½ = 0.5

2/3 = .666…

Integers – No decimals

-3

0

5

Whole Numbers – Non-negative

0

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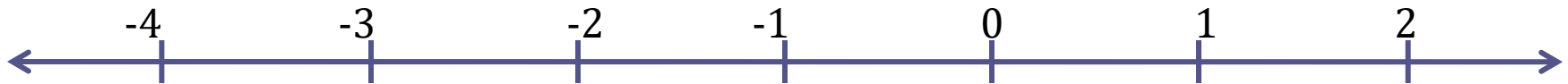
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Irrational Numbers – Cannot be written as fraction

$\sqrt{2} =$ 1.414…

$-\sqrt{5} =$ -2.24…

$\pi =$ 3.14…

1.1 Apply Properties of Real Numbers

- Graph the numbers -0.2, $\frac{7}{10}$, -1, $\sqrt{2}$, -4 on a number line

1.1 Apply Properties of Real Numbers

Property	Addition	Multiplication
Closure	a + b is a real number	ab is a real number
Commutative	a + b = b + a	ab = ba
Associative	(a + b) + c = a + (b + c)	(ab)c = a(bc)
Identity	a + 0 = a	a • 1 = a
Inverse	a + (-a) = 0	a • 1/a = 1
Distributive	a(b + c) = ab + ac	
Definition of Subtraction	a − b = a + (-b)	
Definition of Division	a ÷ b = a • 1/b	

Identify the illustrated property
1. (2 • 3) • 9 = 2 • (3 • 9)
2. 15 + 0 = 15
3. 4(5 + 25) = 4(5) + 4(25)

1.1 Apply Properties of Real Numbers

- Use properties to show that each statement is true and justify each step.
  - $b \cdot (4 \div b) = 4$

$b \cdot \left(4 \cdot \dfrac{1}{b} \right)$ definition of division

$b \cdot \left(\dfrac{1}{b} \cdot 4 \right)$ commutative prop of multiplication

$\left(b \cdot \dfrac{1}{b} \right) \cdot 4$ associative prop of multiplication

$1 \cdot 4$ inverse prop of multiplication

4 identify prop of multiplication

1.1 Apply Properties of Real Numbers

- $3x + (6 + 4x) = 7x + 6$

$3x + (4x + 6)$ commutative prop of addition

$(3x + 4x) + 6$ associative prop of addition

$(3 + 4)x + 6$ distributive prop

$7x + 6$ addition

1.1 Apply Properties of Real Numbers

- Unit analysis
  - When you deal with real-life (word) problems you should check your units to make sure your calculation makes sense
- You work 6 hours and earn $69. What is your earning rate?

- How long does it take to travel 180 miles at 40 miles per hour?

1.1 Apply Properties of Real Numbers

- Convert the following units
  - 150 yards to feet

  - 16 years to seconds

Homework Quiz

- 1.1 Homework Quiz

1.2 Evaluate and Simplify Algebraic Expressions

- Order of Operations

Order	Operation	Examples		
1st	Grouping Symbols	$(2 + 4)$	$\sqrt{2 + 7}$	$\frac{2+5}{3-4}$
2nd	Exponents	2^3		
3rd	Multiply and Divide from left to right	$2 \cdot 6 \div 3 \cdot 4 = 16$		
4th	Add and Subtract from left to right	$2 + 6 - 3 + 4 = 9$		

1.2 Evaluate and Simplify Algebraic Expressions

- Evaluate
  - -2^6

  - $(-2)^6$

  - $5x(x-2)$ when $x = 6$

1.2 Evaluate and Simplify Algebraic Expressions

- Algebraic expressions written as sums

<div align="center">

variable term   constant term

$$2x^3 + 4x - 6$$

coefficients

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- To add or subtract algebraic expressions, combine like terms by adding the coefficients.
  - 2x + (-1) + 5x = 7x – 1

1.2 Evaluate and Simplify Algebraic Expressions

- Simplify
  - ◦ 2n – 1 + 6n + 5

  - ◦ $3p^3 + 5p^2 – p^3$

  - ◦ 8(x – 3) – 2(x + 6)
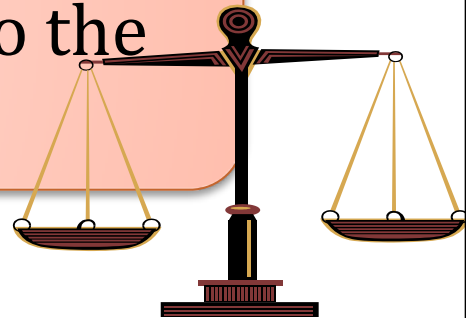
Homework Quiz

- 1.2 Homework Quiz

1.3 Solve Linear Equations

The Golden Rule

Do unto others as you would have them do unto you.

The Golden Rule of Algebra

Do unto one side as you have done unto the other side.

1.3 Solve Linear Equations

- General way to solve linear equations
  - Get the variables all on one side
  - Get everything away from the variables

Always follow the Golden Rule!!!

1.3 Solve Linear Equations

- Solve
  - 4x + 9 = 21

- $-\dfrac{3}{5}x + 1 = 4$

1.3 Solve Linear Equations

- Solve
  - -2x + 9 = 2x – 7

- 3(x + 2) = 5(x + 4)

1.3 Solve Linear Equations

- Solve
  - $\frac{2}{3}x + \frac{5}{6} = x - \frac{1}{2}$

1.3 Solve Linear Equations

- A real estate agent's base salary is $22,000 per year. The agent earns a 4% commission on total sales. How much must the agent sell to earn $60,000 in one year?

Homework Quiz

- 1.3 Homework Quiz

1.4 Rewrite Formulas and Equations

- Common Formulas (Quiz Tomorrow!)

Quantity	Formula
Distance/Rate	d = rt
Temperature	$F = \dfrac{9}{5}C + 32$
Area of a triangle	A = ½ bh
Area of a rectangle	A = ℓw
Perimeter of a rectangle	P = 2ℓ + 2w
Area of a trapezoid	A = ½ (b$_1$ + b$_2$)h
Area of a circle	A = πr^2
Circumference of a circle	C = 2πr

1.4 Rewrite Formulas and Equations

- Solve for a variable
  - Get the variable on one side of the equals sign and everything else (including other variables) on the other side

- Find the radius of a circle with a circumference of 25 feet.

1.4 Rewrite Formulas and Equations

- Solve the perimeter of a rectangle formula for ℓ. Then find the length of a rectangle with width of 7 inches and a perimeter of 30 inches.

1.4 Rewrite Formulas and Equations

- Solve the area of a trapezoid formula for h. Then find h if b_1 = 6 in, b_2 = 8 in, and A = 70 in^2.

- Solve for y
  - $4y - xy = 28$

1.4 Rewrite Formulas and Equations

- A campus bookstore sells T-shirts for $15 each and sweatshirts for $22 each.
  - Write an equation for the bookstore's revenue from selling m T-shirts and n sweatshirts.

  - Solve the equation for m.

Homework Quiz

- 1.4 Homework Quiz

- 1.4 Formula Quiz

1.5 Use Problem Solving Strategies and Models

- Strategies to solve real-life (word) problems
  - Often it is easiest to write an equation in words before you write it in mathese.
  - This is called a verbal model.
  - You think this way in your head already.
- Ways to find a verbal model
  - Use a formula
  - Look for a pattern
  - Draw a diagram

1.5 Use Problem Solving Strategies and Models

- An artic tern flies an average speed of 16.7 miles per hour. How long will it take to fly from its winter grounds in Antarctica to its breeding grounds in Greenland, a distance of 16000 miles?

1.5 Use Problem Solving Strategies and Models

- PARAMOTORING: The table shows the height h of a paramotorist after t minutes. Find the height of the paramotorist after 8 minutes.

Time (min), t	0	1	2	3	4
Height (ft), h	2400	2190	1980	1770	1560

1.5 Use Problem Solving Strategies and Models

- A truck used 28 gallons of gasoline and traveled a total distance of 428 miles. The truck's fuel efficiency is 16 miles per gallon on the highway and 12 miles per gallon in the city. How many gallons of gasoline were used in the city?
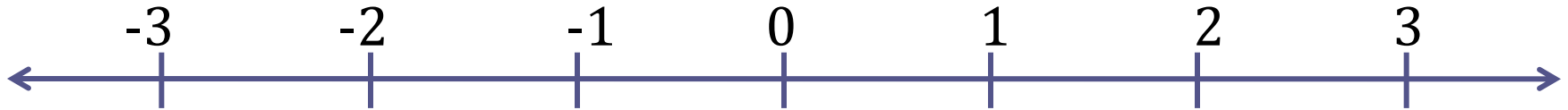
Homework Quiz

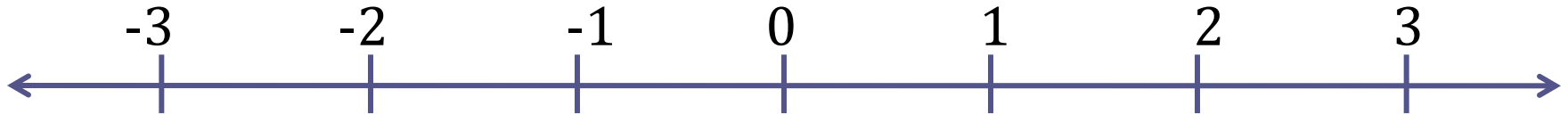- 1.5 Homework Quiz

1.6 Solve Linear Inequalities

- Inequality
  - Similar to an equation, only one side is greater than the other
- Inequalities in one variable can be graphed on a number line
  - Plot the point on the number line
    - Filled dot if equal to; ≤, ≥, =
    - Open dot if not equal to; <, >
  - Draw an arrow or line covering the part of the number line included by the inequality
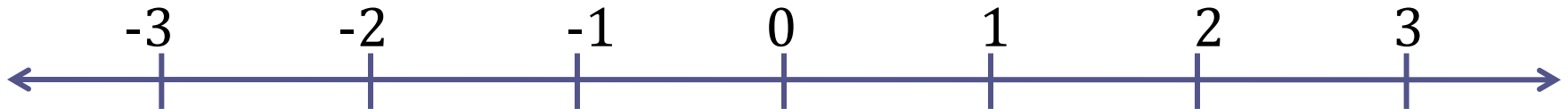
1.6 Solve Linear Inequalities

- Graph x > -2

  -3   -2   -1   0   1   2   3

- Graph -3 ≤ x < 1

  -3   -2   -1   0   1   2   3

- Graph x < 1 or x ≥ 2

  -3   -2   -1   0   1   2   3

1.6 Solve Linear Inequalities

- Solving inequalities
  - Follow the Golden Rule
  - One exception: when you multiply or divide by a negative, reverse the inequality

- 2 < 4
- (-3) 2 < (-3) 4
- -6 🚫 12

1.6 Solve Linear Inequalities

- Solve
  - 4x + 9 < 25

  - 5x – 7 ≤ 6x

1.6 Solve Linear Inequalities

- 1 – 3x ≥ -14

1.6 Solve Linear Inequalities

- Solve
  - -1 < 2x + 7 < 19

- x + 4 ≤ 9 or x − 3 ≥ 7

1.6 Solve Linear Inequalities

- In South Bend, the lowest temperature on record is   -22°F in January, 1943, while the highest temperature on record is 109°F in July, 1934. Write the range of temperatures as an inequality. Then write an inequality giving the temperature range in degrees Celsius.

Homework Quiz

- 1.6 Homework Quiz

1.7 Solve Absolute Value Equations and Inequalities

- Absolute Values

Distance from origin to coordinate

In one dimension, turns the number positive

- | x | = b
  - Distance between x and 0 is b

- | x – k | = b
  - Distance between x and k is b

1.7 Solve Absolute Value Equations and Inequalities

- There are usually two solutions,
  - One for when the expression inside the || is positive
  - One for when the expression inside the || is negative
- Steps to solve
  - Write two equations
    - Absolute value expression +
    - Absolute value expression -
  - Solve each equation
  - Check your solutions

1.7 Solve Absolute Value Equations and Inequalities

- Solve
  - | x – 3 | = 10

  - | 2x + 5 | = 3x

- | 4x – 1 | = 2x + 9

1.7 Solve Absolute Value Equations and Inequalities

- Solve absolute value inequalities the same as equations
  - Exception: write answer as compound inequality

1.7 Solve Absolute Value Equations and Inequalities

- Solve
  - $|2x - 7| > 1$

- $|7 - x| \leq 4$

1.7 Solve Absolute Value Equations and Inequalities

- Ostrich eggs have an average mass of 1950 grams, with a tolerance of 350 grams. Write and solve an absolute value inequality that describes the mass of ostrich eggs.

Homework Quiz

- 1.7 Homework Quiz